CITY SCHOOL OF EDUCATIONAL INSTITUTE, PANDHURNA
OPEN BOOK EXAMINATION 2020-21
Class-12th
Subject- Mathematics

Time – 3 hour   Max. Mark– 100

General Instructions:-
1. All questions are compulsory.
2 Section A is Q.1 to Q.5 are objective type questions. Solve as directed.
Internal choices are given.
3. Section B is from Q.6 to Q.11, each carries 3 marks.
4. Section C is from Q.12 to Q.16, each carries 4marks.
5. Section D is from Q.17 to Q.21, each carries 5 marks.
5. Section E is from Q.22 to Q.23, each carries 6 marks

Section-A

Q.1- Choose the correct option and write

1. The smallest integer function f(x) = [x] is

(a) One-one   (b) Many-one

(c) Both (a) & (b)   (d) None of these

2.

The value of $\tan^{-1}\left(\dfrac{1}{2}\right) + \tan^{-1}\left(\dfrac{1}{3}\right) + \tan^{-1}\left(\dfrac{7}{8}\right)$ is

(a) $\tan^{-1}\left(\dfrac{7}{8}\right)$ (b) $\cot^{-1}(15)$

(c) $\tan^{-1}(15)$ (d) $\tan^{-1}\left(\dfrac{25}{24}\right)$

3.

The value $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is

(a) -7 (b) 7
(c) 8   (d) 10

4. Using determinants, find the equation of the line joining the points (1, 2) and (3, 6).

(a) y = 2x   (b) x = 3y

(c) y = x   (d) 4x – y = 5

5.

$\displaystyle\int \dfrac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}\, dx,\ \dfrac{3\pi}{4} < x < \dfrac{7\pi}{4}$ is equal to

(a) log |sin x + cos x| (b) x

(c) log |x| (d) -x

Q.2-Fill in the blanks-

1. If $f : R \to R$, $g : R \to R$ and $h : R \to R$ are such that $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$, then the value of $(go(foh))(x)$, if $x = 1$ will be_____.

2. If $x^2 + y^2 = 1$, then_____.

3. The number of discontinuous functions $y(x)$ on $[-2, 2]$ satisfying $x^2 + y^2 = 4$ is_____.

4. $\int_1^2 x^2 dx = ---$_____.

5. The area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$ is_____.

Q.3-Match the column –

Column A	Column B
A – A'	nx^{n-1}
$f(x) = [x]$	skew-symmetric
$\int \sin x \, dx$	cos x
d/dx sinx	-cosx+c
d/dx x^n	Many-one

Q.4-Write True/False-

1. If $\tan^{-1} x - \tan^{-1} y = \tan^{-1} A$, then A is equal to $x-y/1+xy$.
2. If A and B are symmetric matrices of the same order, then AB – BA is a symmetric matrix.
3. If $y = (1 + x)(1 + x^2)(1 + x^4).....(1 + x^{2n})$, then the value of dy/dx at $x = 0$ is -1.
4. $\int e^x dx = e^x + C$.
5. d/dx k = 1.

Q.5-Answer in one word/Sentence-

1. If $f(x) = (ax^2 - b)^3$, then the function g such that $f\{g(x)\} = g\{f(x)\}$

2. The value of c in Rolle's Theorem for the function $f(x) = e^x \sin x$, $x \in [0, \pi]$ is

3. If $y = (\tan x)^{\sin x}$, then dy/dx is equal to

4. Evaluate: $\int_0^1 \sin^{-1}(2x/1+x^2)dx$

5. The area common to the ellipses $x^2/a^2+y^2/b^2=1$ and $x^2/b^2+y^2/a^2=1$, $0 < b < a$ is

Section-B

Q.6. Check whether the relations R in **R** defined as R= $\{(a,b):a<=b^3\}$ is reflexive, symmetric or transitive.

Q.7. Find the principal value of $\tan^{-1} (-3^{1/2})$.

Q.8. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Q.9. Evaluate the determinants

|cosx -sinx|

|sinx   cosx|

Q.10. Examine the following functions for continuity:

(a) $(x) = x - 5$

(b) $(x) = 1/x-5, x \neq 5$

(c) $(x) = x^2-25/x+5, x \neq -5$

Q.11.The volume of a cube is increasing at the rate of 8 cm3/s. How fast is the surface area increasing when the length of an edge is 12 cm?

Section - C

Q.12. Find the integral $\int (2x - 3 \cos x + e^x) dx$?

Q.13. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1, x=4$ and the x- axis in the first quadrant. and area of the region bounded by $y^2 = 9x, x= 2, x = 4$ and the x-axis in the first quadrant.

Q.14. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by

(i) $R = \{(a, b) : |a - b|$ is a multiple of 4$\}$

(ii) $R = \{(a, b) : a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

Q.15.. Prove that, $\tan^{-1}1/5 + \tan^{-1}1/7 + \tan^{-1}1/3 + \tan^{-1}1/8 = \pi/4$.

Q.16.. Using properties of determinants, show that

(i) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$

Section - D

Q.17.Differentiate the functions given

$(x+1/x)^x + x^{(1+1/x)}$

Q.18.Find the equation of the normal to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

Q.19.

Integrate the function $\dfrac{\sqrt{\tan x}}{\sin x \cos x}$

Q.20. Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

Q.21. Find the integral of the function $\sin^3(2x+1)$.

Section - D

Q.22.

Find X and Y, if

(i) $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Q.23. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below:

(i) by using product rule

(ii) by expanding the product to obtain a single polynomial

(iii) by logarithmic differentiation

Do they all give the same answer?